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Messages
Mobile Phones
TVs and Flat Panel Display
Streaming Subscriptions
Laptops & Other Appliances
Accessories
Net Cash Balance
Services Revenue
Gross Profit Margin
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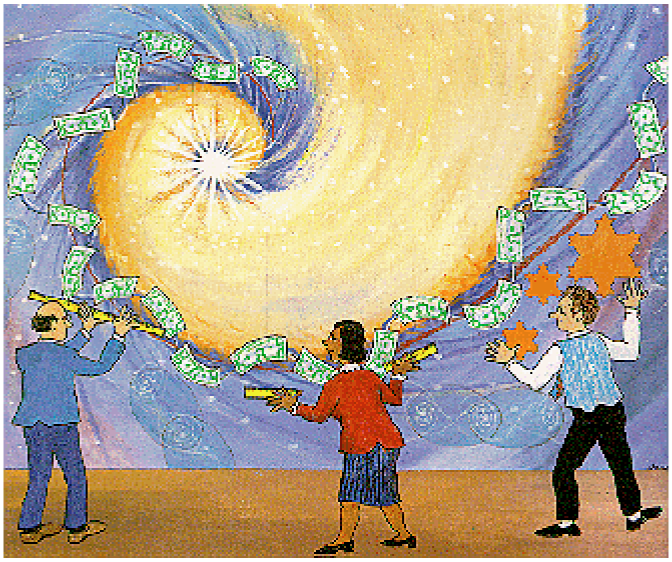
Polarized Fractal Efficiency

by Hans Hannula, Ph.D., C.T.A.



What is fractal geometry, anyway, and how do you use it? Well, you'll find out here. S TOCKS & COMMODITIES *contributor Hans Hannula of MicroMedia describes the construction and use of an indicator derived from fractal geometry, the mathematics that describe chaotic systems.*

Most chaotic systems produce some form of graphic representation. For example, turbulent flow in a stream produces swirls, eddies and vortices. Early chaos researchers found that the triangles, squares, lines and cubes of Euclidean geometry simply did not help in describing, studying or understanding their research problems.

Fortunately, mathematician Benoit Mandelbrot recognized this problem and solved it by describing fractal geometry. As he studied various problems being researched, he realized that many of these problems had in common graphic representations of a very squiggly line. So he asked himself the profound question, "What *is* the dimension of a squiggly line?"

The problem can be represented as shown in Figure 1. A straight line has a dimension of one. A plane surface has a dimension of two. A squiggly line has a dimension between one and two, depending on how much it squiggles. The dimension of the line is not an integer like 1, 2 or 3 but can be a fraction, leading to the term *fractional* or *fractal dimension*. Mandelbrot discovered that many chaotic systems had a constant fractal dimension. Others discovered that systems with the same fractal dimension had other properties in common. Thus, fractal dimension became an important tool in chaos work.

APPLICATION TO PRICE MOTION

Fractal dimensions can be used to study price action in stocks and commodities. Used in a specific way, it can measure how trendy or congested price action is. To understand how to develop such a tool, let us look at a problem that interested Mandelbrot. The question posed was, "How long is a coastline?" The

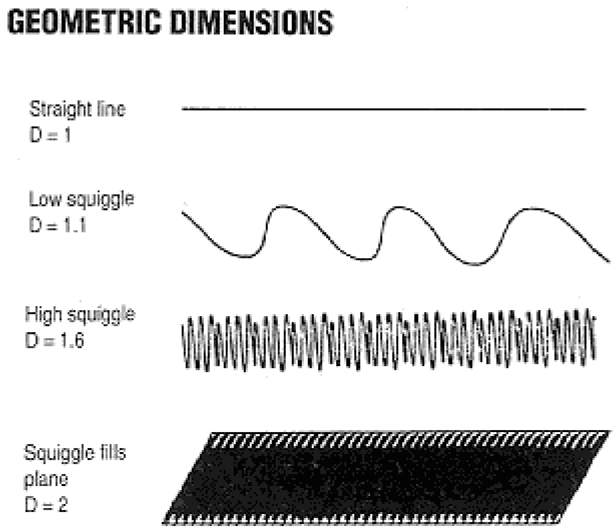


FIGURE 1: GEOMETRIC DIMENSIONS. *What is the dimension of a squiggly line? A straight line has a dimension of one. A plane surface has a dimension of two. A squiggly line has a dimension between one and two, depending on how much it squiggles. The dimension of the line is not an integer line 1, 2 or 3 but can be a fraction, leading to the term fractional or fractal dimension.*

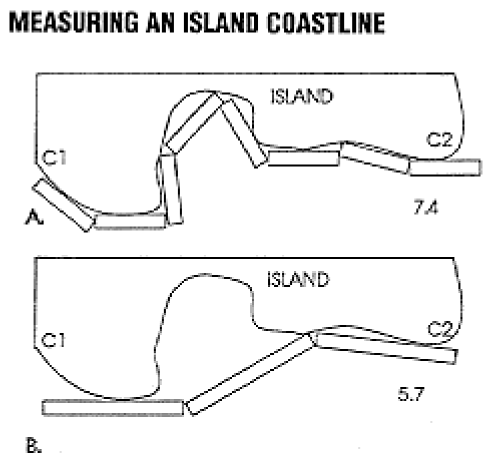


FIGURE 2: MEASURING AN ISLAND COASTLINE. *In A, a short ruler is used, flipped end to end along the coast, while in B, a longer ruler is used. Because the longer ruler does not fit into the coves of the coastline as well, it will meausre a shorter distance. So if one uses shorter and shorter rulers, one can theoretically get longer and longer distances.*

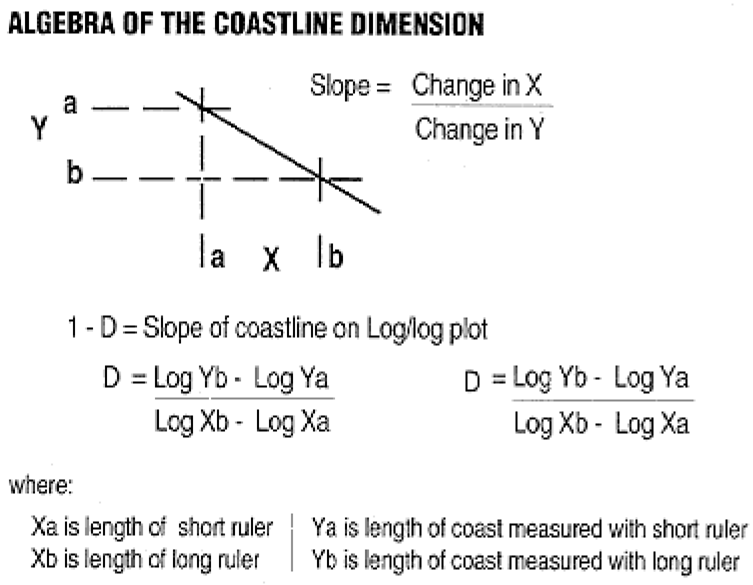


FIGURE 3: ALGEBRA OF THE COASTLINE DIMENSION. *The data used on Figure 2 was plotted on log-log charts, showing that for any given coastline, the relationship between the length of the ruler and the length of the coastline had a constant slope on a log-log chart, leading to the mathematatical relationships shown here.*

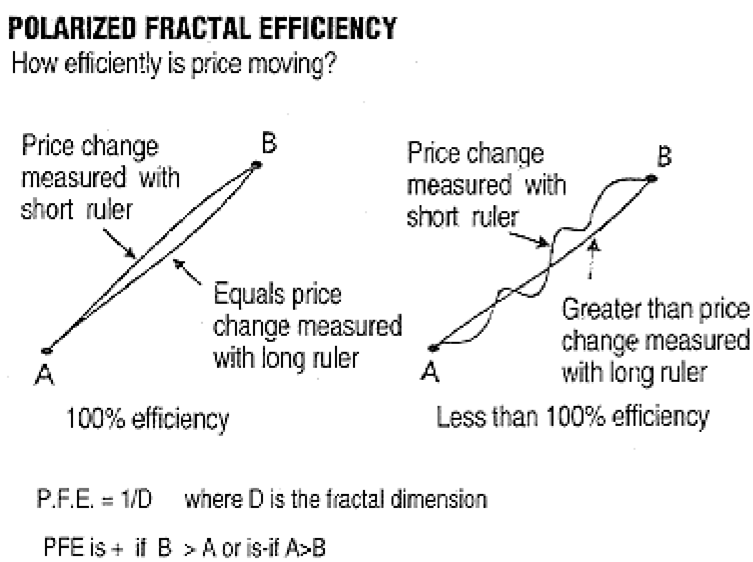


FIGURE 4: POLARIZED FRACTAL EFFICIENCY. *If price motion from A to B is seen, it can move in a straight line. But prices don't do that; they squiggle around, moving with less than 100% efficiency. This efficiency can be measured by dividing the length of the straight line by the length of the squiggly line. But measuring the line is the same problem as the coastline, so we need to use fractal dimension.*

answer, surprisngly, is, "As long as you want to make it!"

To illustrate, look at Figure 2, which depicts two ways to measure an island's coastline. In A, a short ruler is used, flipped end to end along the coast. In B, a longer ruler is used. Because the longer ruler does not fit into the coves of the coastline as well, it will measure a shorter distance. Ruler A might measure 7.4 and ruler B might measure 5.7 in going from point C1 to C2. So if one uses shorter and shorter rulers, one can theoretically get longer and longer distances!

The data on this was plotted on log-log charts, showing that for any given coastline, the relationship between the length of the ruler used and the length of the coastline had a constant slope on a log-log chart. Mandelbrot recognized that this slope was one minus the fractal dimension, leading to the mathematical relationships shown in Figure 3.

PRICES ARE LIKE COASTLINES

A common question is, "How much has price moved?" The expected answer *could* be something along the lines of "Up 2 points." Yet conceptually, the answer could be, "That depends on how I measure it." Such thinking led me to define *polarized fractal efficiency* (PFE) as shown in Figure 4. If one looks at price motion from point A to point B, it can move in a straight line. That is 100% efficient. But prices don't usually do this. They squiggle around, moving with less than 100% efficiency. This efficiency can be measured by dividing the length of the straight line by the length of the squiggly line. But measuring the line is the same problem as the coastline, so we need to use fractal dimension. If we divide the length of the straight line (100% efficient) by the length of the squiggly line, we have a measure of fractal efficiency. Finally, if we attach a plus sign when the move is down and a minus sign when the move is up, we have polarized fractal efficiency.

Figure 5 illustrates measuring the PFE on a stock or commodity bar chart. If we just use the closing prices, we can measure directly from C1 to C2 with a straight line to get distance B, or we can measure along the close-to-close path to get distance A. If distance B was nine and distance A was 25, the efficiency would be 36%. The fractal efficiency equation expresses this efficiency using logarithms. The close-to-close spacing represents our short ruler, while the first-to-last close spacing represents our long ruler.

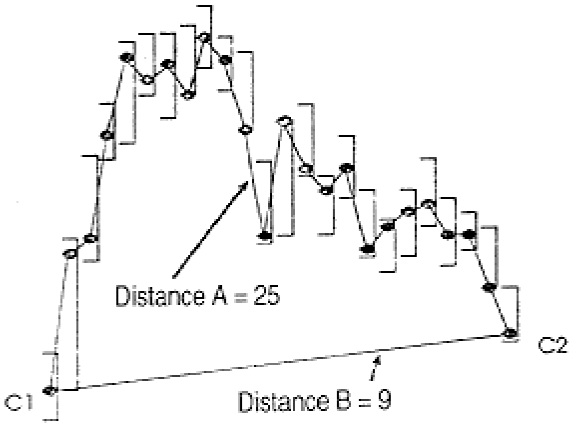
Figure 6 shows the resulting mathematical equation for computing PFE. Each close-to-close line is treated as the hypotenuse of a triangle, the length of which is computed as the square root of the squares of the sides. Adding the close-to-close lengths together gives the length of distance A. Distance B is simply the hypotenuse of the triangle between the first and last close.

APPLYING THE PFE

To apply the PFE, one needs only to decide what number of bars to span and then compute the PFE. Minor smoothing with a five-period exponential moving average (EMA) removes noise caused by the sign changes as trends switch directions. Plotting the filtered PFE under price action provides a measure of how efficiently a market is moving. Figure 7 illustrates the PFE used on the OEX index. I found that a 10-day PFE provides a reasonable compromise between computational delay, which is half the span, and usability of the indicator information.

In Figure 7, note that the PFE tends to have a maximum efficiency of about 43%, either going up or down. This has been found to be true for all stock indices. Other stocks and commodities tend to have slightly

MEASURING PFE ON A CHART



Ignoring logs, efficiency = 9/25 = 36%

FIGURE 5: MEASURING PFE ON A CHART. *If we just use the closing prices, we can measure directly from C1 to C2 with a straight line to get distance B, or we can measure along the close-to-close path to get distance A. If distance B was nine and distance A was 25, the efficiency would be 36%. The fractal efficiency equation expresses this efficiency using logarithms. The close-to-close spacing represents our short ruler, while the first-to-last close spacing represents our long ruler.*

COMPUTING PFE

$$PFE = \frac{[\text{Sign}]\ \sqrt{(Close_n - Close_1)^2 + n^2}}{\sum_{i=2}^{n} \sqrt{(Close_i - Close_{i-1})^2 + 1}}$$

where:
Sign is plus if $CLOSE_n$ is higher than $CLOSE_1$ and minus if $CLOSE_n$ is lower than $CLOSE_1$

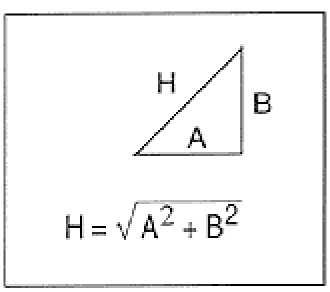


FIGURE 6: COMPUTING PFE. *Here are the resulting mathematical equation for computing Pfe. Each close-to-close line is treated as the hypotenuse of a triangle, the length of which is computed as the square root of the squares of the sides. Adding the close-to-close lengths together gives the length of distance A. Distance B is simply the hypotenuse of the triangle between the first and last close.*

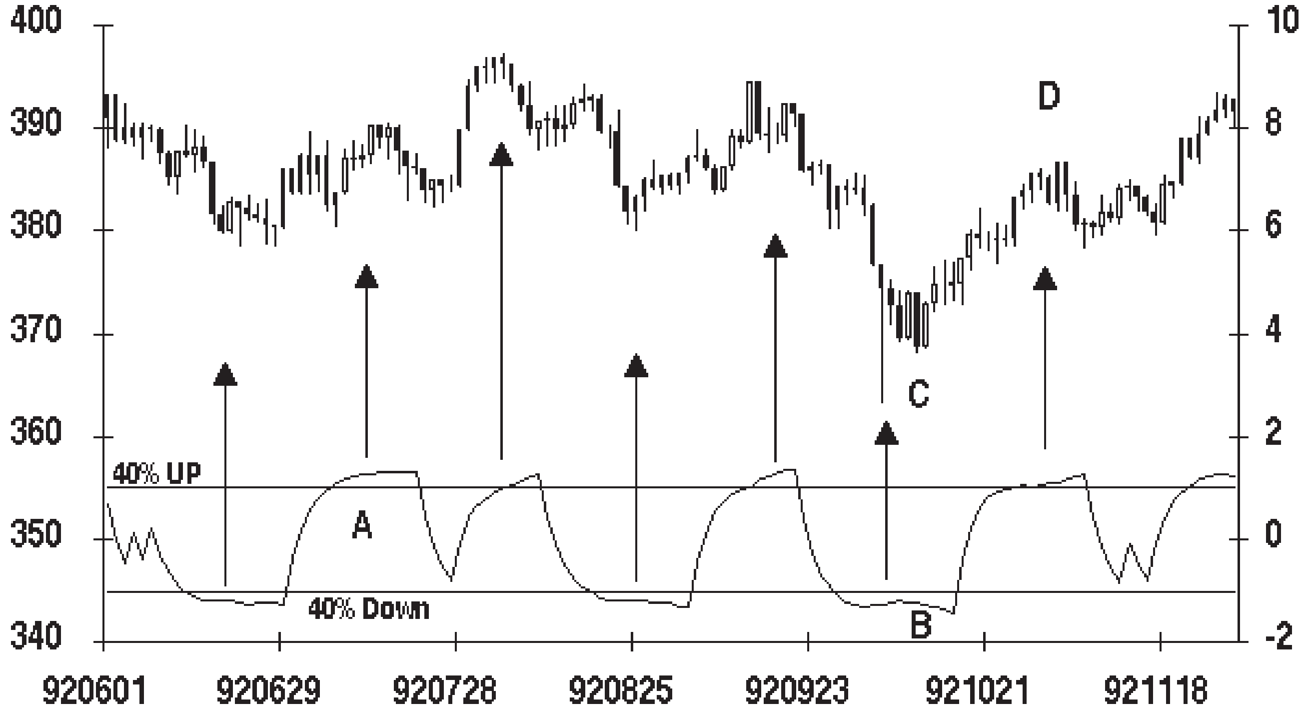


FIGURE 7: OEX AND 10-DAY POLARIZED FRACTAL EFFICIENCY. *PFE tends to have a maximum efficiency of about 43%, either going up or down. This has been found to be true for all stock indices. Other stocks and commodities tend to have slightly different maximums, but the maximum has always been observed. Also note that the transition from trending up to trending down is usually smooth. However, the efficiency may bounce off the zero line or oscillate around it for a time. This middle region is a balance of supply and demand and, thus, a congestion point.*

different maximums, but the maximum has always been observed. Also note that the transition from trending up to trending down is usually smooth. However, the efficiency may bounce off the zero line or oscillate around it for a time. This middle region is a balance of supply and demand and, thus, a congestion point.

PFE also exhibits two other behaviors of interest. Once it passes 66% (for the OEX), it tends to jump sharply to 80%. It seems to lock into being efficient. Often, but not always, the time from this lock-in point to the next top or bottom is a constant time interval, shown by the lock-in points and vertical lines such as the one marked A. This phenomenon suggests that maximum price velocity can only last so long, like a short-distance runner who can only sprint until he runs out of surplus oxygen.

The other behavior useful to traders is that usually right before the end of an efficient period, a hook to maximize efficiency occurs. See point B. This hook usually comes too late in the PFE to be tradable, but it does show up when candlesticks are used. At points C and D, tall candles of opposite colors mark the hooks on the price chart. In this way, the PFE can act as a filter for candlestick patterns.

CAVEAT

Traders should be warned: The PFE measures and reflects only what price has been doing in the past; it does not predict. The safe way to trade the periods of maximum efficiency is to enter by following price action with a trailing stop. Once entered, place a stop above or below the extreme of the hook and watch as the PFE approaches zero. If it crosses cleanly, stay in until PFE reaches maximum efficiency in the other direction. If price congests near zero, exit the trade and then wait for a new maximum efficiency entry.

FRACTAL FINI

Fractal dimension, the new and intriguing way of studying price motion, provides insight into market behavior in ways not found in other indicators. Fractal dimension has been found to be very useful in trading both stocks and commodities. This indicator is one way in which you can begin incorporating knowledge of chaos into your own trading.

Hans Hannula is an engineer, programmer and trader with more than 30 years' market experience. This article is excerpted from his Cash In On Chaos course.

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